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HEAT TRANSFER FROM A SPHERE TO RAREFIED GAS MIXTURES

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Abstract—Measurements of the heat-transfer coefficient from a sphere to rarefied gas mixtures have been made over a range of Knudsen number, Kn, of 0·008–0·4 by utilizing a thermistor. The experimental results on the relation of Nu to Kn and α_{mix} were in good agreement with our analytical equation

$$Nu = \frac{2}{1 + \frac{15}{2}\alpha_{mix}^{-1} . Kn}$$

where

$$Nu = \frac{hD}{\lambda_{mix}}, \text{ Nusselt number;}$$

$$Kn = 85\cdot89 \frac{\mu_{mix}}{Dp}\sqrt{\left(\frac{T}{\bar{M}}\right)}, \text{ Knudsen number;}$$

$$\alpha_{mix} = \frac{\sum_{i=1}^{n} \frac{x_i \alpha_i}{\sqrt{(M_i)}}}{\sum_{i=1}^{n} \frac{x_i}{\sqrt{(M_i)}}}, \text{ accommodation coefficient;}$$

$$\mu_{mix} = \sum_{i=1}^{n} \frac{x_i \mu_i}{\sum_{j=1}^{n} x_j \phi_{ij}}, \text{ viscosity;}$$

$$\lambda_{mix} = \sum_{i=1}^{n} \frac{x_i \lambda_i}{\sum_{j=1}^{n} x_j \phi_{ij}}, \text{ thermal conductivity;}$$

$$\phi_{ij} = \frac{1}{\sqrt{8}}\left(1 + \frac{M_i}{M_j}\right)^{-\frac{1}{2}}\left[1 + \left(\frac{\mu_i}{\mu_j}\right)^{\frac{1}{2}}\left(\frac{M_j}{M_i}\right)^{\frac{1}{4}}\right]^2.$$

NOMENCLATURE

A, area of heat-transfer surface [cm^2];
B, thermistor constant [degK];
Div, divergent operator [dimensionless];
D, characteristic length of thermistor [cm];
f, velocity distribution function [s^3/cm^3];
$Grad$, gradient operator [dimensionless];

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h, heat-transfer coefficient [cal/s cm^2 degK];
I, electric current [A];
j, mechanical equivalent of heat [J/cal];
$\tilde{K}$, proportional constant in Maxwell's inverse fifth power law [cm^6/g s^2];
Kn, Knudsen number [dimensionless];
k, Boltzmann constant [erg/degK];
L_∞, mean free path of gas [cm];
L_{mix}, mean free path of gas mixture [cm];
M, molecular weight [g/mol];
$\bar{M}$, mean molecular weight [g/mol];
m, mass of molecule [g];
Nu, apparent Nusselt number [dimensionless];
Nu^*, actual Nusselt number [dimensionless];
P, pressure [dyne/cm^2], or [mm Hg];
Q, heat transfer from thermistor by any mechanism other than conduction through gas, [cal/s];
q_r, radial heat flux [cal/s cm^2];
R, universal gas constant [erg/mol degK];
R, resistance of thermistor [Ω];
R_∞, resistance of thermistor at temperature T_∞ [Ω];
R_0, radius of sphere [cm];
r, radial distance [cm];
$\bar{r}$, normalized radial distance [dimensionless];
T_w, wall temperature of thermistor [°K];
u, internal energy [erg/molecule];
V, velocity of molecule [cm/s];
X, external force [dyne];
x, mole fraction [dimensionless].

Greek symbols

α, $\cos^{-1} R_0/r$ [dimensionless];
α, accommodation coefficient [dimensionless];
α_{mix}, accommodation coefficient of gas mixtures [dimensionless];
λ_g, thermal conductivity of gas [cal/cm s degK];
λ_{mix}, thermal conductivity of gas mixture [cal/cm s degK];
κ, ratio of internal energy to translational energy, dimensionless;
μ, viscosity of gas [g/cm s];
μ_{mix}, viscosity of gas mixture [g/cm s];
ρ, gas density [g/cm];
ω, integration constant [dimensionless].

INTRODUCTION

MANY studies of conductive heat transfer under rarefied gas conditions have been made by a number of investigators. It seems, however, that only a few investigations, both experimental and analytical, have been devoted to the study of heat transfer from a sphere to rarefied gas or rarefied gas mixtures in the transition regime. Very recently, Springer and Tsai [1], applying Langmuir's model, proposed a new equation for heat conduction through rarefied gases contained between two concentric spheres and made comparison with the experimental results for air by Takao and for helium by Peterson.

In 1962, Lees and Liu [2] made an analytical study of the conductive heat transfer from a fine wire on the basis of the kinetic theory of gases. Their solution was achieved by solving the Maxwell's transport equation on the assumption that the velocity distribution function can be approximated by the two-sided Maxwellian distribution function. Their method could be extended to spherical geometry. Hurlbut [3] extended the Lees and Liu's analysis to include arbitrary values of accommodation coefficient α, because the analysis of Lees and Liu was carried out only for the case of complete accommodation.

This paper presents the experimental results on the conductive heat transfer from a sphere to a few kinds of pure gases and their mixtures and also proposes its analytical solution, which was in good agreement with the experimental results. It was found that the relation between Nusselt number and Knudsen number is the same form as that for a cylinder.

EXPERIMENTAL INVESTIGATION

Experimental determination of Nusselt number is based on the measurement of the resistance of a nearly spherical thermistor, placed in a rarefied gas mixture at various pressures. The rate of generation of heat in the thermistor under a steady state, is equal to the rate of dissipation from the thermistor. This becomes

$$(jRI^2 - Q) = hA(T - T_\infty). \tag{1}$$

The relation between R and T for a thermistor is expressed by

$$R = R_\infty \exp\left[B\left(\frac{1}{T} - \frac{1}{T_\infty}\right)\right] \tag{2}$$

Thus, the expression for the Nusselt number is given as follows:

$$\left.\begin{aligned} Nu &= \frac{hD}{\lambda_{\text{mix}}} = \frac{(jRI^2 - Q)}{\pi T_\infty \lambda_{\text{mix}} D} \\ &\qquad \times \frac{[1 - \sigma \ln (R_\infty/R)]}{\sigma \ln (R_\infty/R)} \\ \sigma &= \frac{T_\infty}{B}. \end{aligned}\right\} \tag{3}$$

The relation between Nu and Kn can be obtained by measuring the resistance at various gas pressures under the condition of constant heating current.

A balanced type bridge circuit was used to measure the resistance of the thermistor R as shown in Fig. 1. The thermistor employed in the measurement was produced by the Gow-Mac Instrument Company in U.S.A. [4], and has about 7000 Ω resistance at room temperature. Its form is beads coated by glass as small as about 0·5 mm in diameter, and inserted into the cylindrical hole of diameter 6 mm, drilled in $50 \times 50 \times 50$ mm^3 cell block made from brass (Fig. 2). A line power source of a.c. 100 V was stabilized through a commercial magnetic voltage regulator. It was supplied to the high precision d.c. voltage regulator. The circuit indicated in Fig. 3 is the same one as designed by Takahei [5]. The output of about 350 V d.c. is supplied to the constant d.c. current regulator. The circuit shown in Fig. 3 was designed by referring to the circuit proposed by Shimoda [6]. The output current to the bridge was chosen to be 3·8 mA.

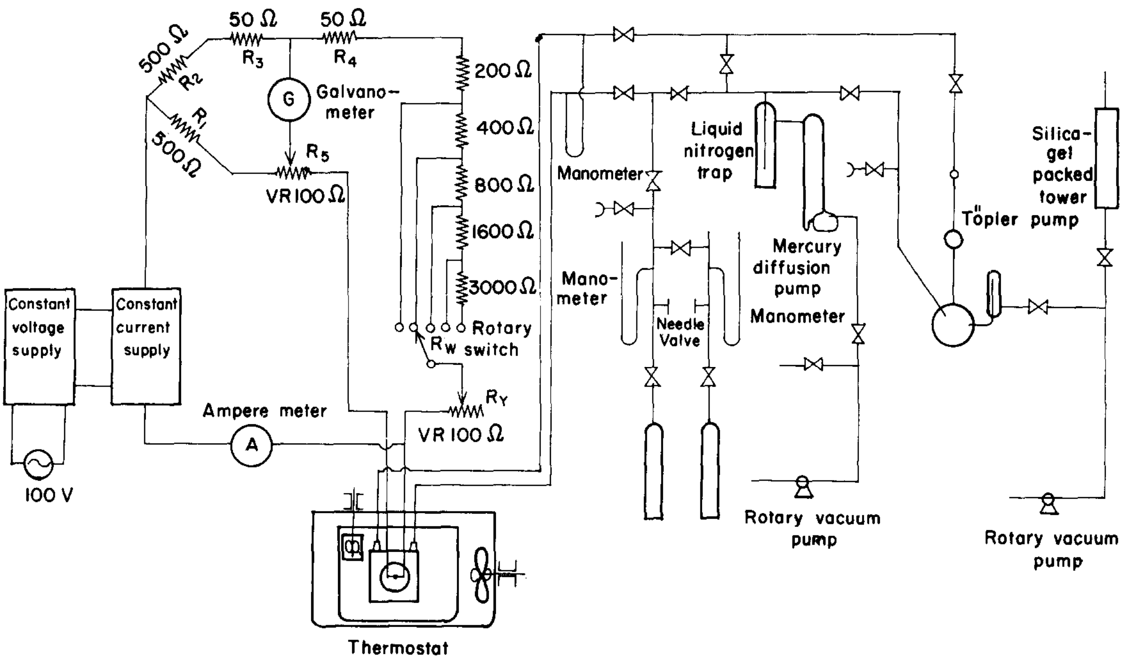

FIG. 1. Diagram of experimental apparatus.

The ripple was attenuated to a degree not appreciable by a galvanometer and by a milliammeter. Furthermore, the output current can be kept constant even if the variation of the resistance of the thermistor brought large variation of the load impedance.

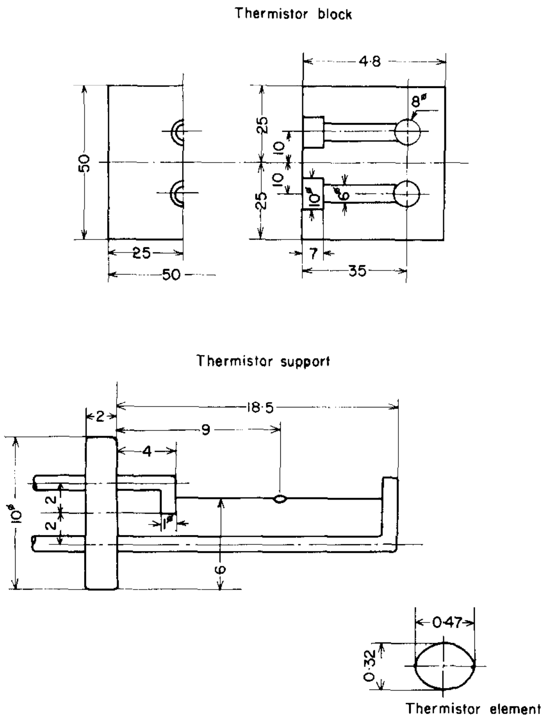

FIG. 2. Schematic diagram of cell block and thermistor.

A gas mixture of known concentration in a sample bulb was admitted into the cylindrical hole in the cell block by means of a Töpler pump after the system was evacuated by a mercury diffusion pump and a rotary vacuum pump. The gas was compressed to the pressure of about 18 mm Hg and isolated from the vacuum system by keeping the stopcock 1 closed. Then the gas pressure was measured by a di-octylphtalate oil manometer to an accuracy of 0·1 mm Hg. Then the electric current was supplied to the thermistor and maintained for several minutes prior to the measurement of the resistance, in order to insure the attainment of steady state condition, which can be confirmed from the zero deflection of the galvanometer.

The cell block was placed in a thermostat. The temperature was regulated to maintain 31·5°C and the oscillation of the temperature was reduced to $\frac{1}{1000}$ degC.

The details are described in our report of "Analysis of two component gas mixture at low pressure by a thermistor-actuated thermal conductivity cell" *Bulletin of The Tokyo Institute of Technology* **69**, 39–47 (1965).

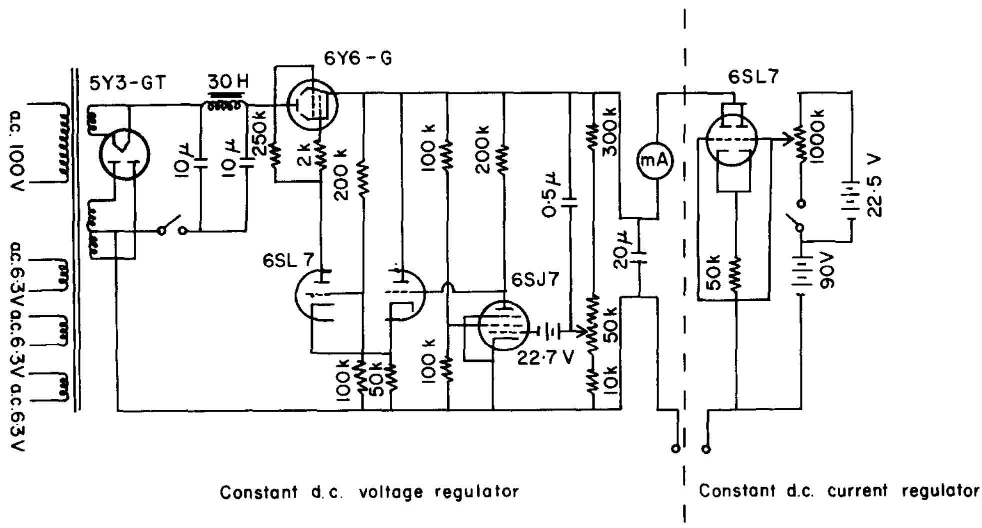

FIG. 3. Electronic circuit of constant a.c. voltage regulator and constant d.c. current regulator.

RESULTS OF EXPERIMENT

The thermistor constant B was determined in the following way. Firstly, the resistance of the thermistor was measured for various electric currents and the extrapolated value at zero current was taken as the true resistance at the thermostat temperature. Secondly, the same procedure was repeated changing the thermostat temperature. The relation between R/R_∞ and $(T_\infty - T)/T_\infty T$ was obtained as shown in Fig. 4. Thus, the value of constant B was determined to be 3500 degK from Fig. 4.

The effect of air pressure on the thermistor resistance was measured to confirm its reproducibility as indicated in Fig. 5. The results obtained before and after a series of experiments with hydrogen and nitrogen gas mixtures were quite satisfactory.

The relation between the resistance of the thermistor and the pressure for H_2–N_2 gas mixtures is shown in Fig. 6, in which the concentration was taken as a parameter. The same relation for He–N_2 gas mixtures is shown in Fig. 7.

From these results, it seems possible to obtain the relation between Nusselt number and Knudsen number with the help of equation (3). However, it is necessary to take into account the effect of thermal resistance of the coated glass on the thermistor, for hydrogen or helium, because of their high thermal conductivity.

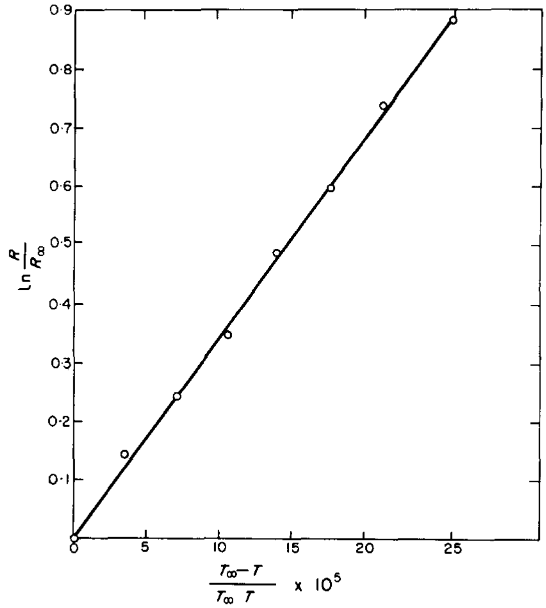

FIG. 4. Variation of resistance of thermistor with temperature.

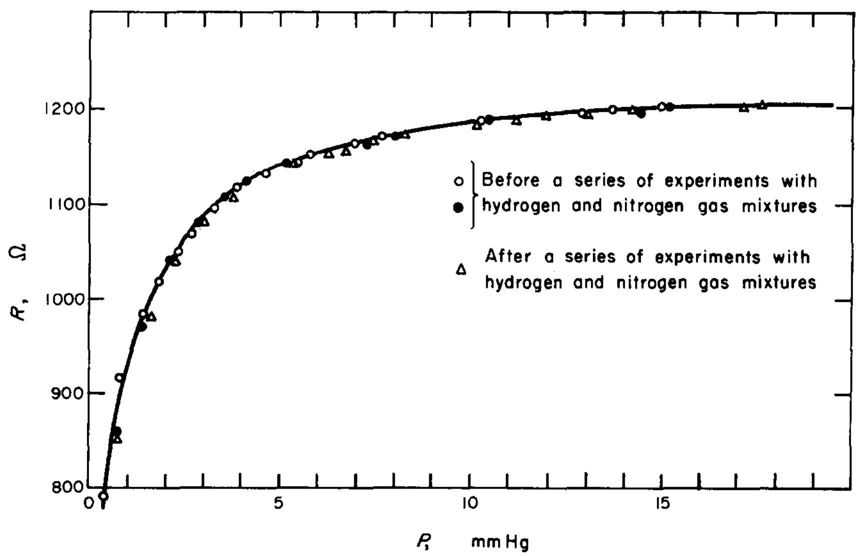

FIG. 5. Reproducibility test by air.

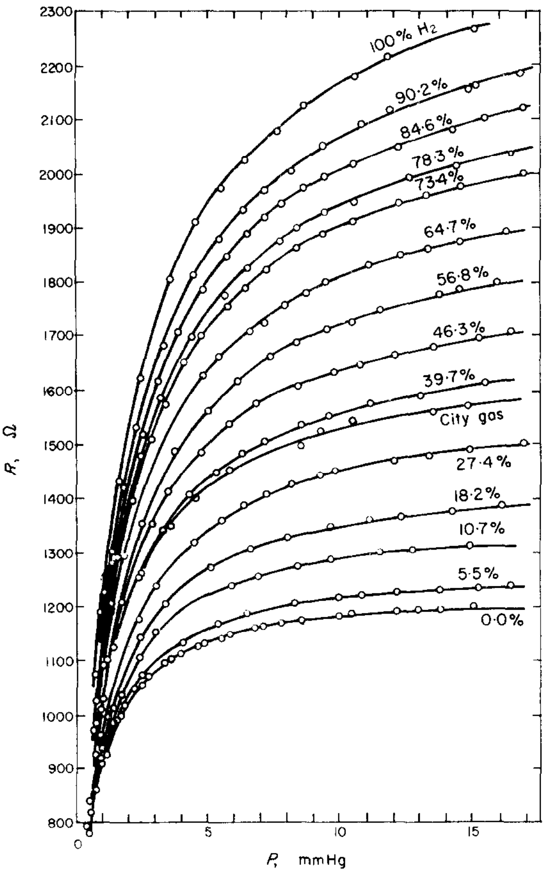

FIG. 6. Relation between resistance of thermistor and pressure for H_2–N_2 gas mixtures of various concentrations with values of concentration as a parameter.

The correction for Nusselt number due to this effect can be made. The apparent Nusselt number was evaluated as a function of Knudsen number by using the experimental results with the aid of equation (3). The thermal conductivity of a gas mixture λ_{mix} at the surface temperature of the thermistor can be evaluated from the experimental results presented in reference [7] and from the method of Mason and Saxena [8]

$$\lambda_{mix} = \sum_{i=1}^{n} \frac{x_i \lambda_i}{\sum_{j=1}^{n} x_j \phi_{ij}} \tag{4}$$

in which x_i is mole fraction and λ_i is the thermal conductivity of the pure component i. The coefficient ϕ_{ij} is given as follows:

$$\phi_{ij} = \frac{1}{\sqrt{8}}\left(1 + \frac{M_i}{M_j}\right)^{-\frac{1}{2}}\left[1 + \left(\frac{\mu_i}{\mu_j}\right)^{\frac{1}{2}} \times \left(\frac{M_j}{M_i}\right)^{\frac{1}{4}}\right]^2. \tag{5}$$

The thermal conductivity data at a given temperature are available from the table in references [7, 9]. The surface temperature was approximated by the thermistor temperature.

The characteristic length D was determined from the following equation which means that πD^2 is equal to the surface area of a spheroidal thermistor;

$$\pi D^2 = 2\pi b^2 \left\{1 + \left(\frac{a}{b}\right)^2 \frac{1}{\sqrt{[1 - (a/b)^2]}} \times \ln \frac{1 + \sqrt{[1 - (a/b)^2]}}{a/b}\right\} \tag{6}$$

where a, b denote short radius and long radius respectively, and D was 0·0419 cm in this experiment.

The heat loss Q, that is, the heat conduction from the thermistor through platinum wire leads 27·6 μ in diameter and 10 mm in total length, (see Fig. 2) was calculated from the following simple expression on the assumption that the heat conduction from the wire to the gas can be neglected. The above assumption is valid if the wire is placed in an infinite medium.

$$Q = 2\pi\delta^2\lambda_M \frac{T - T_\infty}{L} \tag{7}$$

in which δ, λ_M, L denote the radius of platinum wire, the thermal conductivity of platinum and the length of platinum wire, respectively. Corrections for radiation and free convection were neglected because the maximum value of the correction in the present investigation were about 0·1 per cent, and 1 per cent, respectively.

Knudsen number for pure gas and gas mixtures was calculated from the following expression at the wall temperature of the cell

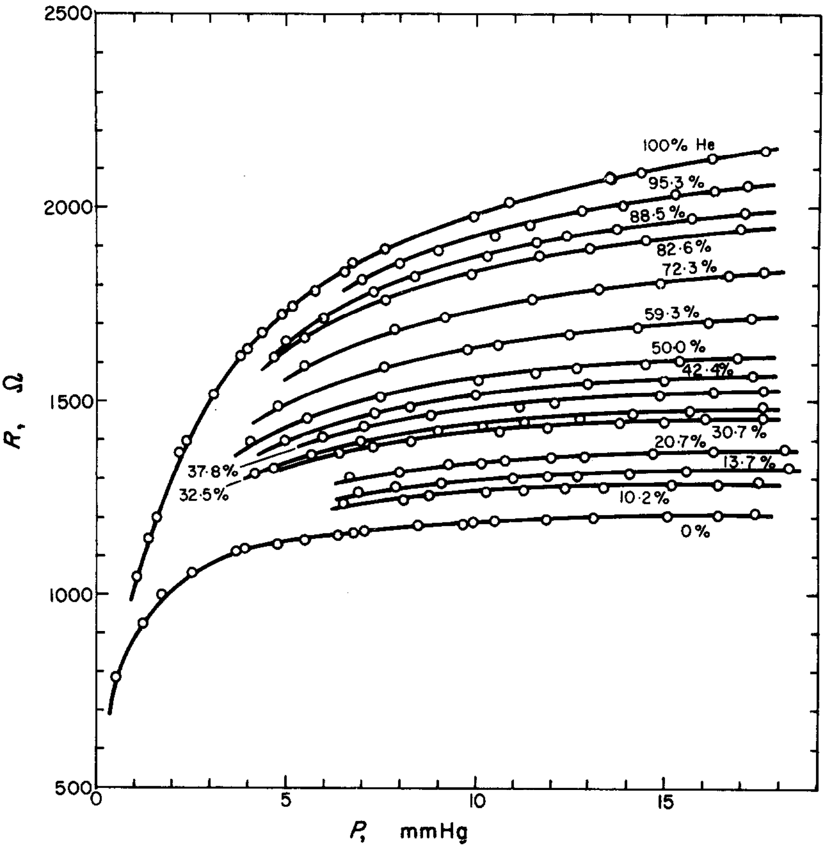

FIG. 7. Relation between resistance and pressure for He–N_2 gas mixtures of various concentrations with values of concentration as a parameter.

block T_∞ [10]

$$Kn = 85{\cdot}89\,\frac{\mu_{\text{mix}}}{Dp}\sqrt{\left(\frac{T_\infty}{\overline{M}}\right)} \tag{8}$$

where μ_{mix} = viscosity of gas mixture

$\overline{M} = M_1x_1 + M_2x_2$; mean molecular weight.

The viscosity of gas mixtures is obtained from the analogous equation to the previously given expression for the thermal conductivities, [8] or from the experimental results presented in reference [7];

$$\mu_{\text{mix}} = \sum_{i=1}^{n} \frac{x_i\,\mu_i}{\sum\limits_{j=1}^{n} x_j\,\phi_{ij}} \tag{9}$$

$$\phi_{ij} = \frac{1}{\sqrt{8}}\left(1 + \frac{M_i}{M_j}\right)^{-\frac{1}{2}}\left[1 + \left(\frac{\mu_i}{\mu_j}\right)^{\frac{1}{2}} \times \left(\frac{M_j}{M_i}\right)^{\frac{1}{4}}\right]^2. \tag{10}$$

The actual Nusselt number Nu^* can be obtained by considering of the heat conduction in the glass region surrounded by two concentric spheres, that is,

$$Nu = \frac{Nu^*}{1 + Nu^*\gamma\lambda_{\text{mix}}} \tag{11}$$

$$\gamma = \frac{s_2 - s_1}{2\,s_1\lambda_s} \tag{12}$$

where λ_s: the thermal conductivity of glass.

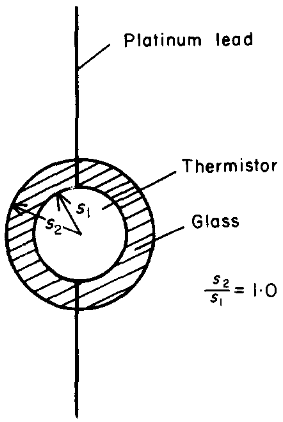

FIG. 8. Schematic diagram of thermistor element.

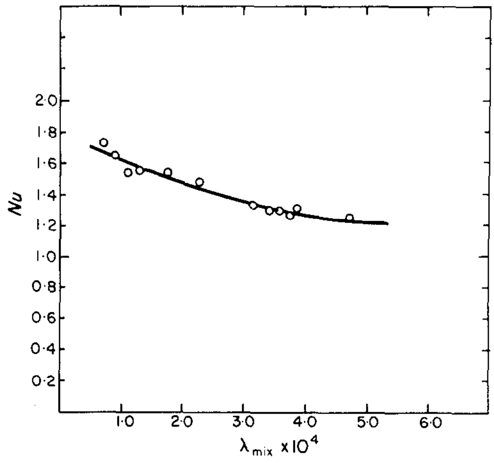

FIG. 9. Apparent Nusselt number as a function of thermal conductivity of gas.

s_1, s_2 denote the radii of inner and outer spheres respectively (see Fig. 8).

From equation (11) it is found that Nu depends on λ_{mix} as follows:

$$Nu = \frac{c_1}{1 + c_2 \lambda_{mix}} \tag{13}$$

Figure 9 indicates Nu as a function of λ_{mix} for various gas mixtures when $Kn = 0{\cdot}01$. The constants c_1 and c_2 appearing in equation (13) were determined from these results to be 1·79 and 0·103 respectively. The solid line in Fig. 9 expresses the relation between Nu and λ_{mix} calculated from equation (13) with these constants. Thus the actual Nusselt number was obtained as follows:

$$Nu^* = \frac{Nu}{1 - 0{\cdot}0577 . 10^4 \lambda_{mix} Nu} \tag{14}$$

Figure 10 gives the Nu^* for various gas mixtures and pure gas in the form of a function of Kn.

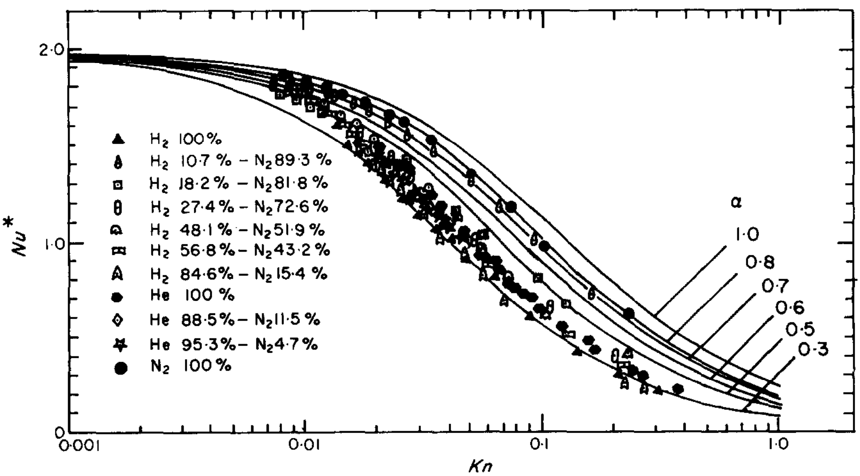

FIG. 10. Relation between Nu^* and Kn for various gas mixtures.

AN ANALYSIS OF CONDUCTIVE HEAT TRANSFER FROM A SPHERE TO RAREFIED GAS

In this section, Nusselt number was calculated analytically as a function of Knudsen number on the basis of the same procedure of Lees and Liu as mentioned in the introduction.

This procedure is to solve the Maxwell integral equation of transport utilizing the two-sided Maxwellian distribution function. Maxwell's equation for a physical quantity ψ which depends on the velocity of a molecule is given as follows: [7, 11]

$$\frac{\partial n\overline{\psi}}{\partial t} + \mathrm{div}_T\, n\overline{\psi V} - n\frac{\overline{\partial \psi}}{\partial t} + \overline{V\,.\,grad_T\,\psi} + \frac{X}{m}.\overline{grad_V\,\psi} = \Delta\psi \quad (15)$$

where a bar script denotes the mean quantity by averaging over all velocity space, and $\Delta\psi$, is the change of ψ produced by collisions.

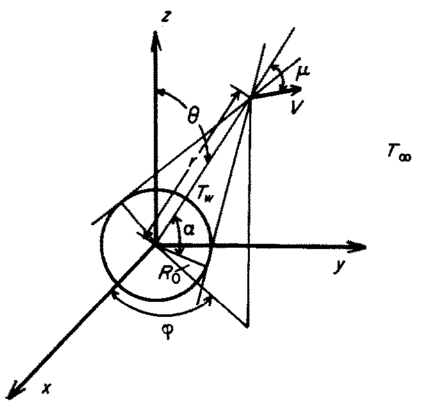

FIG. 11. Diagram for a sphere in the spherical polar coordinate system.

We consider a sphere of radius R_0 placed in an infinite gaseous media of single component at an arbitrary pressure. The temperature on the surface is set T_w, and the temperature at infinity, T_∞ (see Fig. 11). In the spherical coordinate system, equation (15) can be written in the following form:

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\int f V_r \psi\,\mathrm{d}V + \frac{1}{r\tan\theta}\int f V_\theta \psi\,\mathrm{d}V + \int f\left(\frac{1}{r}V_\varphi V_r + \frac{1}{r\tan\theta}V_\varphi V_\theta\right)\frac{\partial\psi}{\partial V_\varphi}\mathrm{d}V + \int f\left(\frac{1}{r}V_\theta V_r - \frac{1}{r\tan\theta}V_\varphi^2\right)\frac{\partial\psi}{\partial V_\theta}\mathrm{d}V - \int f\left(\frac{1}{r}V_\theta^2 + \frac{1}{r}V_\varphi^2\right)\frac{\partial\psi}{\partial V_\varphi}\mathrm{d}V = \Delta\psi. \quad (16)$$

By applying Lee's model, the velocity distribution function was assumed as follows:

$$f_1 = n_1\left(\frac{\beta_1}{\pi}\right)^{\frac{3}{2}}\exp\left[-\beta_1 V^2\right], \text{ or}$$

$$f_2 = n_2\left(\frac{\beta_2}{\pi}\right)^{\frac{3}{2}}\exp\left[-\beta_2 V^2\right] \quad (17)$$

according as

$$0 \leqslant \mu \leqslant \frac{\pi}{2} - \alpha, \text{ or } \frac{\pi}{2} - \alpha \leqslant \mu \leqslant \pi$$

where $n_1, n_2, \beta_1 = m/2kT_1$ and $\beta_2 = m/2kT_2$ are unknown functions of radial distance. Setting $\psi = m, mv_r, \frac{1}{2}mv^2 + u$ or $(\frac{1}{2}mv^2 + u)v_r$, we obtain the following ordinary differential equations from equations (16) (17), on the assumption that internal energy, u is independent of molecular velocity.

$$\bar{n}_1\bar{T}_1^{\frac{1}{2}} = \bar{n}_2\bar{T}_2^{\frac{1}{2}} \quad (18)$$

$$\bar{n}_1\bar{T}_1^{\frac{3}{2}} - \bar{n}_2\bar{T}_2^{\frac{3}{2}} = \omega \quad (19)$$

$$\sin^3\alpha\frac{\mathrm{d}}{\mathrm{d}\bar{r}}(\bar{n}_1\bar{T}_1 - \bar{n}_2\bar{T}_2) - \frac{\mathrm{d}}{\mathrm{d}\bar{r}}(\bar{n}_1\bar{T}_1 + \bar{n}_2\bar{T}_2) = 0 \quad (20)$$

$$\sin^3\alpha\frac{\mathrm{d}}{\mathrm{d}\bar{r}}(\bar{n}_1\bar{T}_1^2 - \bar{n}_2\bar{T}_2^2) - \frac{\mathrm{d}}{\mathrm{d}\bar{r}}(\bar{n}_1\bar{T}_1^2 + \bar{n}_2\bar{T}_2^2) = \frac{4}{15}\frac{R_0}{L_\infty}\frac{1}{\bar{r}^2}\left[\bar{n}_1(1 - \sin\alpha) + \bar{n}_2(1 + \sin\alpha)\right]\omega \quad (21)$$

where all quantities are expressed non-dimensionally by characteristic number density n_∞, temperature T_∞, and the radius of the sphere R_0, and all dimensionless quantities are denoted by a bar superscript, and ω is an integration constant.

The change of ψ produced by collisions $\Delta\psi$, was expressed from Maxwell's inverse fifth power force law

$$F = m_1 m_2 \frac{\tilde{K}}{r^5}. \quad (22)$$

Then the mean free path, L_∞ and the viscosity, μ_∞ become [2]

$$L_\infty = \frac{1}{3A_2\rho_\infty}\left(\frac{\pi k T_\infty}{\tilde{K}}\right)^{\frac{1}{2}} \quad (23)$$

$$\mu = \frac{kT}{\frac{3}{2}A_2(2m\tilde{K})^{\frac{1}{2}}} \quad (24)$$

where $A = 1{\cdot}3682$ and

$$\Delta(\tfrac{1}{2}mV^2 + u)V_r = p/\mu(-\tfrac{2}{3}q_r^{(T_r)}) \quad (25)$$

where P = total pressure, and $q_r^{(T_r)}$ = radial translational energy flux.

$$p = \tfrac{1}{2}k\,[n_1T_1(1 - \sin\alpha) + n_2T_2(1 + \sin\alpha)] \quad (26)$$

$$q_r^{(T_r)} = \tfrac{1}{2}\pi^{-\frac{1}{2}}m\cos^2\alpha(n_1\beta_1{}^{-\frac{3}{2}} - n_2\beta_2{}^{-\frac{3}{2}}) = kn_\infty T_\infty\left(\frac{2k}{\pi m}\right)^{\frac{1}{2}}\omega\left(\frac{R_0}{r}\right)^2. \quad (27)$$

The boundary condition at $\bar{r} = 1$ and at $\bar{r} = \infty$ are

$$\frac{T_1 - T_2}{T_w - T_2} = \alpha; \qquad T = T_2 = T_\infty, \qquad n = n_2 = n_\infty \quad (28)$$

where α denotes the accommodation coefficient. In the present investigation, the amount of maximum departure of T_w/T_∞ from unity was only about 0·2, so that it will be valid to assume that

$$\left.\begin{aligned} \bar{n}_1 &= 1 + N_1 \\ \bar{n}_2 &= 1 + N_2 \\ \bar{T}_1 &= 1 + t_1 \\ \bar{T}_2 &= 1 + t_2 \\ N_1, N_2, &t_1, t_2 \geqslant 1. \end{aligned}\right\} \quad (29)$$

Upon substitution of equations (29) into (18) (19) (20) and (21), one obtains the following set of linearized equations:

$$N_1 + \tfrac{1}{2}t_1 = N_2 + \tfrac{1}{2}t_2 \quad (30)$$

$$(N_1 - N_2) + \tfrac{3}{2}(t_1 - t_2) = \omega \quad (31)$$

$$\sin^3\alpha\frac{d}{d\bar{r}}(N_1 + t_1 - N_2 - t_2) - \frac{d}{d\bar{r}}(N_1 + t_1 + N_2 + t_2) = 0 \quad (32)$$

$$\sin^3\alpha\frac{d}{d\bar{r}}(N_1 - N_2 + 2t_1 - 2t_2) - \frac{d}{d\bar{r}}(2 + N_1 + N_2 + 2t_1 + 2t_2) \simeq \tfrac{8}{15}\frac{R_0}{L_\infty}\frac{1}{\bar{r}^2}\omega. \quad (33)$$

The boundary condition at infinity becomes

$$\bar{r} \to \infty; \qquad t_2 = 0, \quad N_2 = 0. \quad (34)$$

From equation (30) and equation (31),

$$t_1 - t_2 = \omega \quad (35)$$

$$N_1 - N_2 = -\tfrac{1}{2}\omega. \quad (36)$$

Then by integrating equation (32),

$$N_1 + N_2 + t_1 + t_2 = \text{const.} = \tfrac{1}{2}\omega. \quad (37)$$

With the aid of the above equation and the boundary condition at infinity, equation (33) is integrated to give

$$t_1 + t_2 = \tfrac{8}{15}\frac{R_0}{L_\infty}\omega\frac{1}{\bar{r}} + \omega. \quad (38)$$

Hence,

$$t_1 = \omega\left(1 + \tfrac{4}{15}\frac{R_0}{L_\infty}\frac{1}{\bar{r}}\right) \quad (39)$$

$$t_2 = \tfrac{4}{15}\frac{R_0}{L_\infty}\omega\frac{1}{\bar{r}} \quad (40)$$

$$\bar{T}_1 = 1 + t_1 = 1 + \omega\left(1 + \tfrac{4}{15}\frac{R_0}{L_\infty}\right) \quad (41)$$

$$\bar{T}_2 = 1 + t_2 = 1 + \tfrac{4}{15}\frac{R_0}{L_\infty}\omega. \quad (42)$$

The boundary condition at $\bar{r} = 1$ is

$$\frac{\bar{T}_1 - \bar{T}_2}{\bar{T}_w - \bar{T}_2} = \alpha. \tag{43}$$

From equation (41), (42) and (43), there follows

$$\omega = \frac{\alpha(\bar{T}_w - 1)}{\alpha\frac{4}{15}\frac{R_0}{L_\infty} + 1}. \tag{44}$$

At limit of $Kn \to 0$,

$$\omega_{Kn \to 0} = \tfrac{15}{4}\frac{L_\infty}{R_0}\frac{T_w - T_\infty}{T_\infty}. \tag{45}$$

We may assume for the radial heat flux q_r

$$q_r = (1 + \theta\kappa)q_r^{(T_r)} \tag{46}$$

where κ denotes the ratio of internal energy to translational energy and $\kappa = [5 - 3\gamma/3(\gamma - 1)]$, θ is a pure number and nearly unity. Therefore, the radial heat flux in the continuum regime, $q_{r,Kn\to 0}$ becomes from equation (27)

$$q_{r, Kn \to 0} = (1 + \theta\kappa)\, kn_\infty \left(\frac{2k}{\pi m}\right)^{\frac{1}{2}} \frac{R_0}{r^2} \times \tfrac{15}{4} L_\infty(T_w - T_\infty). \tag{47}$$

The above expression readily yields the Fourier law of heat conduction,

$$q_{r, Kn \to 0} = \lambda_q \frac{R_0}{r^2}(T_w - T_\infty),$$
$$\lambda_g = (1 + \theta\kappa)\, \lambda_g^{(M)} \tag{48}$$

as we utilize the relation of $\lambda_g^{(M)} = \frac{15}{4}\mu k/m$ for a monatomic gas and equations (23), (24).

As Nu is expressed from the definition,

$$Nu = \frac{hD}{\lambda_g} = \frac{D}{\lambda_g}\frac{q_{r,\bar{r}=1}}{T_w - T_\infty} = \frac{D}{\lambda_g}\frac{q_{r,Kn\to 0,\bar{r}=1}}{T_w - T_\infty} \times \frac{q_{r,\bar{r}=1}}{q_{r,Kn\to 0,\bar{r}=1}} = \frac{D}{\lambda_g}\frac{\omega}{\omega_{Kn\to 0}} q_{r,Kn\to 0,\bar{r}=1} \tag{49}$$

we eventually obtain the following expression for Nusselt number as a function of Kn and α with the aid of equations (44), (45) and (47):

$$Nu = \frac{2}{1 + \frac{15}{2}Kn\alpha^{-1}}, \qquad Kn = \frac{L_\infty}{2R_0}. \tag{50}$$

COMPARISON BETWEEN ANALYTICAL AND EXPERIMENTAL RESULTS

The above relation with various values of α are shown by solid lines in Fig. 10. The results for some pure gases were in good agreement with equation (47) as indicated in Fig. 10, where the values of α for hydrogen, helium and nitrogen were chosen as 0·3, 0·35 and 0·8 respectively. It seems that these values of the accomodation coefficients are reasonable in comparison with the values obtained by other investigators, as given in Table 1 (references [1, 10, 12] and the tables presented in the survey report on accomodation coefficient by Hartnett [13]).

Table 1. Accommodation coefficient of several gases at 300°K [1, 10, 12]

	Grilly	Amdur	Thoms	Gregory	Kennard	Petersen
O_2–Pt	0·80	0·80			0·62 0·82	
N_2–Pt					0·68 0·81	
He–Pt	0·50	0·40	0·25		0·38 0·50	
He–glass					0·32 (130°C)	0·30
H_2–Pt	0·40	0·30	0·20	0·30		
H_2–glass					0·36	

For a multi-component gas mixture, the accommodation coefficient will be expressed by the following equation from the energy balance

$$\alpha_{\text{mix}} = \frac{\sum \frac{x_i \alpha_i}{\sqrt{m_i}}}{\sum \frac{x_i}{\sqrt{m_i}}} \tag{51}$$

where α_i is the accomodation coefficient for pure gas, and x_i is a mole fraction.

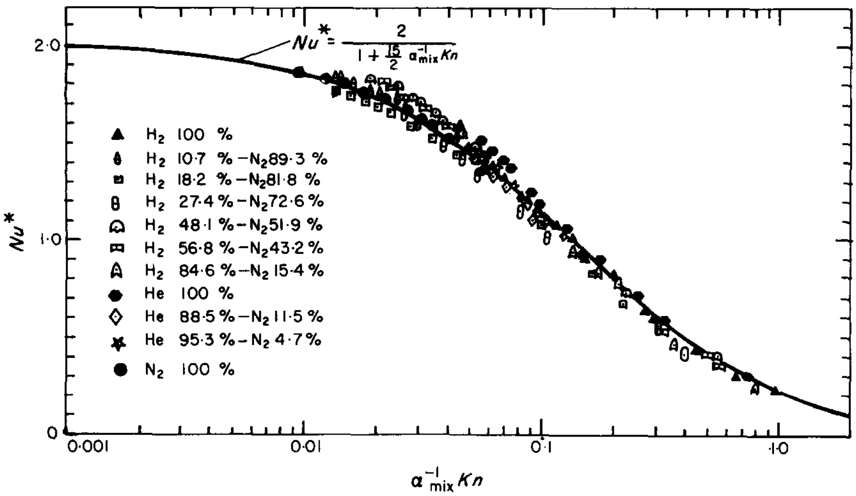

FIG. 12. Relation between Nu^* and $Kn\ \alpha_{\text{mix}}^{-1}$.

Figure 12 indicates the relation between Nu^* and $\alpha^{-1}\ Kn$ for various gas mixtures, in which α_{mix} is evaluated from equation (50).

CONCLUSION

Experimental study of the conductive heat transfer from a sphere in various rarefied gas mixtures were in good agreement with the analytical equation (49) obtained for pure Maxwellian gas, in which Nu correlates with $\alpha_{\text{mix}}^{-1}\ Kn$ uniquely.

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Résumé—On a mesuré le coefficient de transport de chaleur d'une sphère dans des mélanges gazeux raréfiés dans une gamme de nombre de Knudsen Kn de 0,008 à 0,4 en utilisant une thermistance. Les résultats expérimentaux sont en bon accord avec notre équation théorique reliant Nu à Kn et à α_{mix}:

$$Nu = \frac{2}{1 + (15/2)\,\alpha_{mix}^{-1} \cdot Kn}$$

où:

$$Nu = \frac{hD}{\gamma_{mix}}, \text{ est le nombre de Nusselt;}$$

$$Kn = 85{,}89\,\frac{\mu_{mix}}{Dp}\sqrt{\left(\frac{T}{\overline{M}}\right)}, \text{ le nombre de Knudsen;}$$

$$\alpha_{mix} = \frac{\sum_{i=1}^{n} \frac{x_i \alpha_i}{\sqrt{(M_i)}}}{\sum_{i=1}^{n} \frac{x_i}{\sqrt{(M_i)}}}, \text{ le coefficient d'accommodation;}$$

$$\mu_{mix} = \sum_{i=1}^{n} \frac{x_i \mu_i}{\sum_{j=1}^{n} x_j \phi_{ij}}, \text{ la viscosité;}$$

$$\lambda_{mix} = \sum_{i=1}^{n} \frac{x_i \lambda_i}{\sum_{j=1}^{n} x_j \phi_{ij}}, \text{ la conductivité thermique;}$$

$$\phi_{ij} = \frac{1}{\sqrt{8}}\left(1 + \frac{M_i}{M_j}\right)^{-\frac{1}{2}}\left[1 + \left(\frac{\mu_i}{\mu_j}\right)^{\frac{1}{2}}\left(\frac{M_j}{M_i}\right)^{\frac{1}{4}}\right]^2.$$

Zusammenfassung—Messungen des Wärmeübergangskoeffizienten von einer Kugel an verdünnte Gasgemische wurden unter Anwendung eines Thermistors über einen Bereich der Knudsenzahl von $Kn = 0{,}008$–$0{,}4$ durchgeführt. Die Versuchsergebnisse für die Beziehung von Nu zu Kn und α_{mix} standen in guter Übereinstimmung mit unserer analytischen Gleichung

$$Nu = \frac{2}{1 + (15/2)\,\alpha_{mix}^{-1} \cdot Kn}$$

mit

$$Nu = \frac{hD}{\lambda_{mix}} \text{ Nusseltzahl}$$

$$Kn = 85{,}89\,\frac{\mu_{mix}}{Dp}\sqrt{\left(\frac{T}{\overline{M}}\right)} \text{ Knudsenzahl}$$

$$\alpha_{mix} = \frac{\sum_{i=1}^{n} \frac{x_i \alpha_i}{\sqrt{(M_i)}}}{\sum_{i=1}^{n} \frac{x_i}{\sqrt{(M_i)}}}, \quad \text{Angleichsbeiwert}$$

$$\mu_{mix} = \sum_{i=1}^{n} \frac{x_i \mu_i}{\sum_{j=1}^{n} x_j \phi_{ij}}, \quad \text{Zähigkeit}$$

$$\lambda_{mix} = \sum_{i=1}^{n} \frac{x_i \lambda_i}{\sum_{j=1}^{n} x_j \phi_{ij}}, \quad \text{Wärmeleitfähigkeit}$$

$$\phi_{ij} = \frac{1}{\sqrt{8}} \left(1 + \frac{M_i}{M_j}\right)^{-\frac{1}{2}} \left[1 + \left(\frac{M_i}{M_j}\right)^{\frac{1}{4}} \left(\frac{M_j}{M_i}\right)^{\frac{1}{4}}\right]^2.$$

Аннотация—С помощью термистора проведены измерения коэффициента переноса тепла от шара к разреженным газовым смесям в диапазоне значений критерия Кнудсена 0,008–0,4. Экспериментальные данные по зависимости Nu от Kn и α_{mix} хорошо согласуются с нашим аналитическим уравнением
где

$$Nu = \frac{2}{1 + (15/2)\, \alpha_{mix}^{-1} \,.\, Kn}$$

где

$$Nu = \frac{hD}{\lambda_{mix}} \quad \text{критерий Нуссельта,}$$

$$Kn = 85{,}89 \frac{\mu_{mix}}{Dp} \sqrt{\left(\frac{T}{\bar{M}}\right)} \quad \text{критерий Кнудсена,}$$

$$\alpha_{mix} = \frac{\sum_{i=1}^{n} \frac{x_i \alpha_i}{\sqrt{(M_i)}}}{\sum_{i=1}^{n} \frac{x_i}{\sqrt{(M_i)}}}, \quad \text{коэффициент аккомодации,}$$

$$\mu_{mix} = \sum_{i=1}^{n} \frac{x_i \mu_i}{\sum_{j=1}^{n} x_j \phi_{ij}}, \quad \text{коэффициент вязкости,}$$

$$\lambda_{mix} = \sum_{i=1}^{n} \frac{x_i \lambda_i}{\sum_{j=1}^{n} x_j \phi_{ij}}, \quad \text{коэффициент теплопроводности,}$$

$$\phi_{ij} = \frac{1}{\sqrt{8}} \left(1 + \frac{M_i}{M_j}\right)^{-\frac{1}{2}} \left[1 + \left(\frac{\mu_i}{\mu_j}\right)^{\frac{1}{2}} \left(\frac{M_j}{M_i}\right)^{\frac{1}{4}}\right]^2.$$